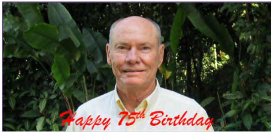
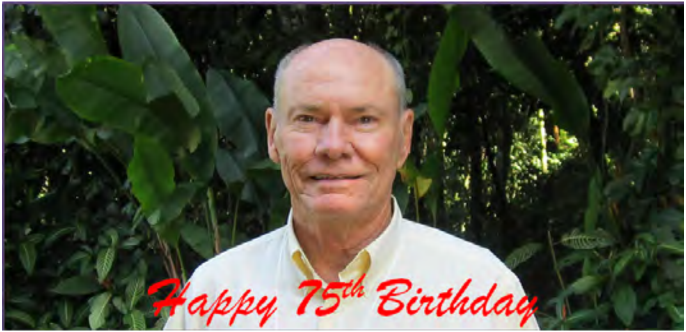


Compressive Hyperspectral Imaging of Space Objects

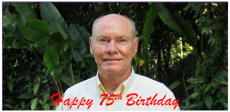
Peter Zhang and Robert J. Plemmons

Wake Forest University

Happy Birthday, Bob!



I'm very grateful to Bob that the research grants headed by him have been able to support me in the last seven years. I'm also grateful that we have published much together, because without his name, I might as well forget about it.



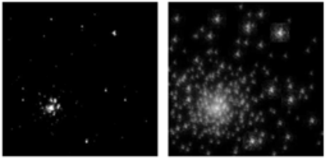
Imaging through Atmospheric Turbulence

Example of an observed blurred starfield image is given by

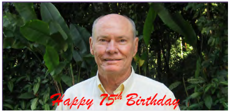
$$g(x, y) = h(x, y) * f(x, y) + \epsilon(x, y), \tag{1}$$

where h is an essentially unknown blurring kernel, f is the true image to reconstruct and ϵ is i.i.d. noise.

$$h = \left| \mathcal{F}^{-1} \left(p e^{i\phi(x,y)} \right) \right|^2, \tag{2}$$



Bardsley, Jefferies, Nagy, and Plemmons, (2006).



Hyperspectral Imaging (HSI) through Atmosphere

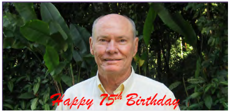
The acquired image at wavelength λ is

$$g_\lambda(x,y) = h_\lambda(x,y) * f(x,y,\lambda) + \epsilon_\lambda(x,y), \tag{3}$$

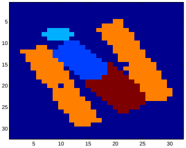
where the blurring kernel is

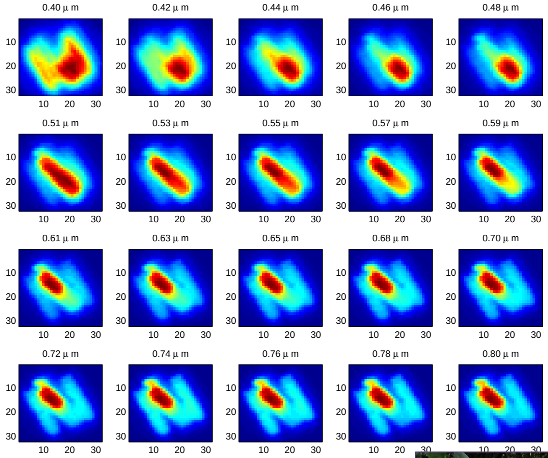
$$h_\lambda = \left| \mathcal{F}^{-1}\left(p e^{i(\frac{2\pi}{\lambda}\phi)} \right) \right|^2, \tag{4}$$

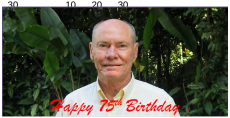
where ϕ is the phase function assumed to be known via wavefront sensing, or through estimation by multiframe blind deconvolution (MFBD) techniques.



Simulated HSI Images through Atmosphere

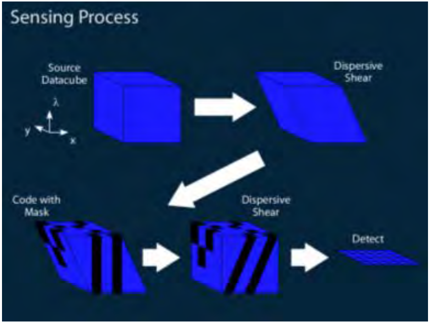




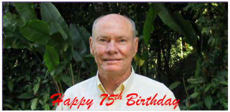


Compressive Sensing in HSI

A Double-Disperser Coded-Aperture Snapshoft Spectral Imager (DD-CASSI).

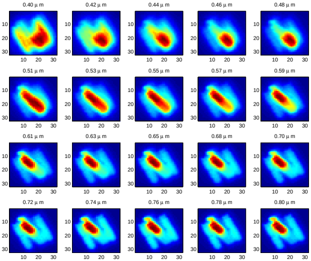


$$g(x,y) = \int_{\lambda} C_{\lambda}(x,y) f(x,y,\lambda) d\lambda. \quad (5)$$

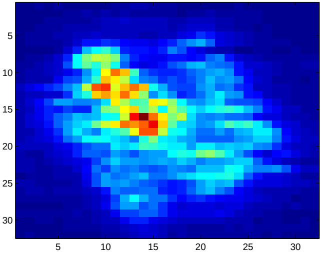


Compressive Sensing in HSI

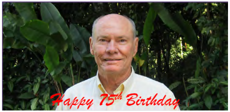
A single snapshot of blurred hyperspectral images by DD-CASSI.



$\Rightarrow$



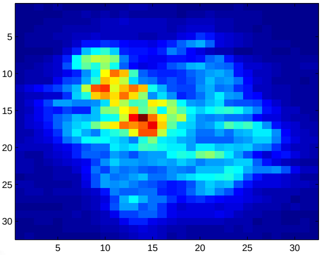
Note: due to the multiplexing, the DD-CASSI image is worse than the least-severely blurred image at the longest wavelength, but much better than the most-severely blurred image at the shortest wavelength.



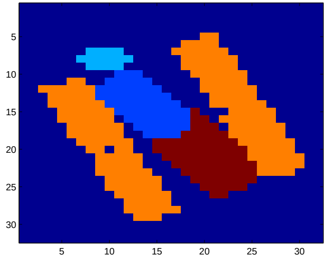
Reconstruction from Compressed Measurements

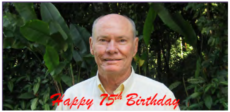
From a single or a few DD-CASSI images of space objects through atmospheric turbulence, we would like to know the following:

- The true hyperspectral object.
- Possibly also the blurring kernel, or more specifically, the phase function.



$\Rightarrow$





Problem Formulation

A single snapshot of hyperspectral images degraded by atmospheric turbulence is

$$g(x, y) = \int_{\lambda} C_{\lambda}(x, y)[h_{\lambda}(x, y) * f(x, y, \lambda)]d\lambda + \epsilon(x, y). \tag{6}$$

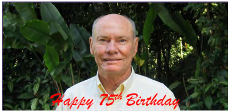
where $C_{\lambda}(x, y)$ is the wavelength-dependent coded aperture.

If we discretize the problem, we can write it in a linear form,

$$g = Af + \epsilon, \tag{7}$$

where $A = (A_1, A_2 \ldots, A_d)$, and $A_{\lambda} = \text{diag}(c_{\lambda})H_{\lambda}$, a $n^2 \times n^2$ square matrix. d is the number of spectral channels. Due to the multiplication by $\text{diag}(c_{\lambda})$, we can no longer compute the operations of A_{λ} through FFT techniques.

<u>Note</u>: matrix A often has poor compressive sensing properties. Interestingly, adding the blurring actually improves these properties, i.e., removing H_{λ} from A_{λ} would make the properties of A even worse.

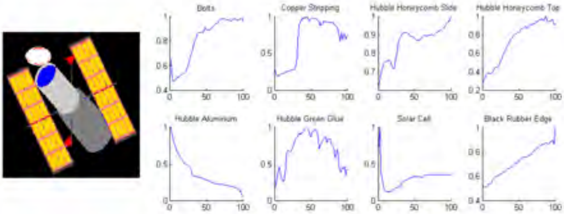


A Low-Rank Solution Model: Spectral Unmixing

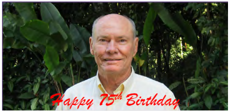
The linear spectral unmixing model:

$$f(x, y, \lambda) = \sum_{i=1}^{m} s_i(\lambda) u_i(x, y). \tag{8}$$

- A decomposed form of solution and hence much fewer parameters.
- Positivity and sum-to-one supports are enforced.



Z., Wang, Pauca and Plemmons, (2008).



Segmentation Model

The parameter-saving segmentation model:

$$f(x, y, \lambda) = \sum_{i=1}^{m} s_i(\lambda) u_i(x, y). \tag{9}$$

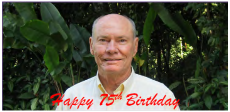
And then the system model looks like

$$g = \int_{\lambda} C_{\lambda}(x, y)[h_{\lambda}^{\phi}(x, y) * \sum_{i=1}^{m} s_i(\lambda) u_i(x, y)] d\lambda + \epsilon(x, y). \tag{10}$$

Knowns: a coded, blurred and noisy image g, and .

Unknowns: the phase function ϕ, spectral signatures $s_{i\lambda}$, and support functions, u_i.

Approach: we take a two-step approach, that is, we estimate the phase function first and then the hyperspectral object.

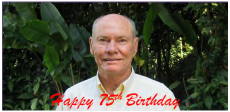


Estimate Phase Function via a Reference Star

If we know the spectrum of a single reference star, then the only unknown is the phase function, which can be estimated using the following regularized and convex functional.

$$J(\phi) = \frac{1}{2} \left\| \int_{\lambda} C_{\lambda}(x,y)[h_{\lambda}^{\phi}(x,y) * s(\lambda)u(x,y)]d\lambda - g \right\|_2^2 + \frac{\alpha}{2} \langle \mathcal{A}^{-1}\phi, \phi \rangle. \tag{11}$$

where we assume ϕ is a second-order stationary process with zero mean, and is characterized by its auto-covariance operator $\mathcal{A}$.



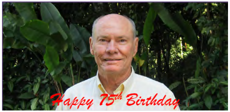
Estimate Phase Function via a Reference Star

The convex functional $J(\phi)$ can be optimized by most optimization algorithms. Here, we use the limited-memory BFGS algorithm developed by Bardsley, Nagy,Jefferries, and Plemmons (2006), which only needs the gradient of the functional.

We can derive the derivative of J against ϕ, similarly as in Vogel, Chan and Plemmons (1997),

$$\mathrm{J}'(\phi) = 2\sum_{\lambda} \mathrm{Imag}[\tilde{\mathrm{H}}_{\lambda}^{*}\mathcal{F}(\tilde{\mathrm{h}}_{\lambda}\mathrm{Real}(\mathcal{F}^{-1}(\mathrm{F}_{\lambda}^{*}\mathrm{R}))], \tag{12}$$

where $\tilde{\mathrm{H}}_{\lambda} = \mathrm{p}e^{\mathrm{i}\frac{2\pi}{\lambda}\phi}$, $\tilde{\mathrm{h}} = \mathcal{F}^{-1}(\tilde{\mathrm{H}})$, $\mathrm{F}_{\lambda} = \mathcal{F}(\mathrm{f}_{\lambda})$, and $\mathrm{R} = \mathcal{F}(\mathrm{C}_{\lambda}\mathrm{r})$, where r is the residual function.

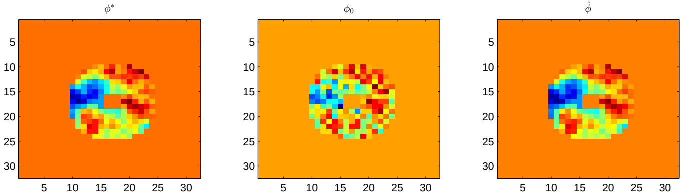


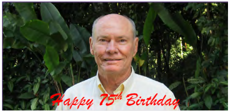
Estimated Phase Function

Here, we simulate the phase function using the von Karman phase spectrum,

$$P(x, y) = \sqrt{.023}(D/r_0)^{5/6}(x^2 + y^2)^{-11/6}. \tag{13}$$

and $\phi = P^{1/2}(x, y)W(x, y)$, where W is the white complex noise. Due to the random nature of the phase function, we assume the initial guess is taken from a space centered around the true phase function, but with a large radius.



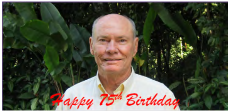


Estimation of the Segmented Hyperspectral Object

From now on, we assume the phase function is known as the estimated $\hat{\phi}$. In the next step, we can estimate the support functions u_i and spectral signatures s_i.

$$
\begin{aligned}
J(u,s) &= \frac{1}{2}\left\| \int_\lambda C_\lambda(x,y)\left[h_\lambda(x,y) * \sum_{i=1}^{m} s_i(\lambda)u_i(x,y)\right] - g(x,y)\right\|_2^2 \\
&+ \alpha \sum_{i=1}^{m}\int_{\mathbb{R}^2}\sqrt{\nabla_x^2 u_i + \nabla_y^2 u_i} + \frac{\beta}{2}\sum_{i=1}^{m}\int_\lambda s_i^2(\lambda), \qquad (14)
\end{aligned}
$$

where we have the total variation regularization for u_i and the Tikhonov regularization for s_i.



Estimation Using the Alternating Approach

Let $u = (\mathrm{vec}(u_1)^T, \ldots, \mathrm{vec}(u_m)^T)^T$, and $s = (s_1^T, \ldots, s_m^T)^T$.

At the j^{th} iteration,

1. Given $s^{(j-1)}$, solve for the membership functions $u^{(j)}$.

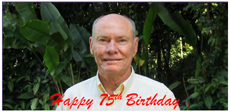
$$\min_{u} TV(u), \ \text{s.t. } A_s^{(j-1)} u = g, \tag{15}$$

where $A_s^{(j-1)}$ is a $1 \times m$ block matrix, in which the i^{th} block is $\sum_\lambda \mathrm{diag}(c_\lambda) H_\lambda s_{i\lambda}^{(j-1)}$ with size $n^2 \times n^2$.

2. Given $u^{(j)}$, solve for the spectral signatures, $s^{(j)}$.

$$\min_{s} \|g - A_u^{(j)} s\|_2^2 + \beta \|s\|_2^2, \tag{16}$$

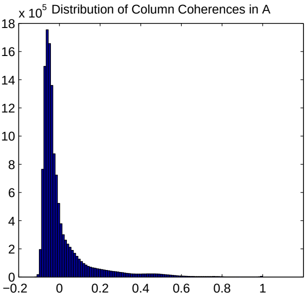
where each column of $A_u^{(j)}$ is $\mathrm{diag}(c_\lambda) H_\lambda u_i$.

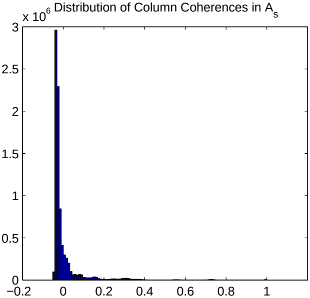


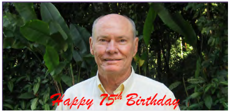
Benefits of the Soluion Model: Mutual Incoherence

Before we solve for u, we first compare some matrix properties of A_s related to the compressive sensing with those of the original system matrix A.

First, we compare the distribution of column correlations or coherences.





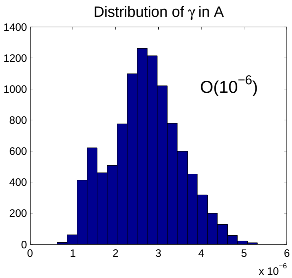


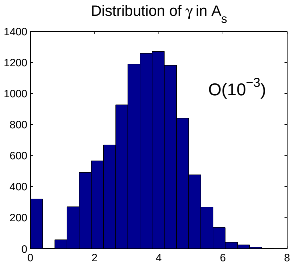
Benefits of the Soluion Model: Restricted Eigenvalue

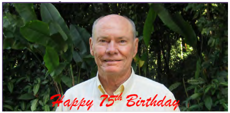
Definition

A matrix $X \in \mathbb{R}^{n \times p}$ satisfies the restricted eigenvalue condition over a subset S with parameters $\alpha \geq 1$ and $\gamma > 0$ if

$$\frac{\|X\triangle\|_2}{\sqrt{n}} \geq \gamma\|\triangle\|_2 \text{ for all } \triangle \in \mathbb{R}^p \text{ such that } \|\triangle_{S_c}\|_1 \leq \alpha\|\triangle_S\|_1. \tag{17}$$







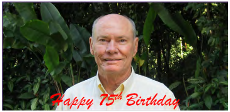
Alternating Direction Method of Multipliers (ADMM)

Introducing an auxillary variable v:

$$\min_{\mathrm{v}} \mathrm{TV}(\mathrm{v}), \text{ s.t. } \mathrm{A_s u} = \mathrm{g} \text{ and } \mathrm{u} = \mathrm{v},, \tag{18}$$

Rewriting with augmented Lagrangian multipliers:

$$\mathrm{L}(\mathrm{u},\mathrm{v},\mathrm{w}) = \|\mathrm{A_s u} - \mathrm{g}\|_2^2 + \beta \mathrm{TV}(\mathrm{v}) + \mathrm{w}^{\mathrm{T}}(\mathrm{u}-\mathrm{v}) + \frac{\alpha}{2}\|\mathrm{u}-\mathrm{v}\|^2. \tag{19}$$

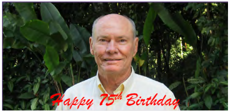


Support Function Estimation Using the ADMM

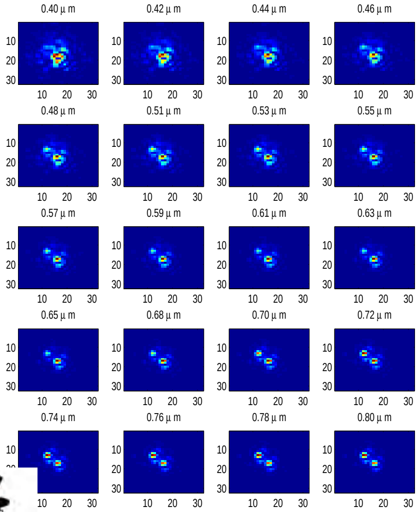
Iteratively, we estimate u, v and w, in the following order,

$$
\begin{aligned}
\mathrm{u}^{\mathrm{k}+1} &= (\mathrm{A}_{\mathrm{s}}^{\mathrm{T}}\mathrm{A}_{\mathrm{s}} + \alpha \mathrm{I})^{-1}\left[\mathrm{A}_{\mathrm{s}}^{\mathrm{T}}\mathrm{g} + \alpha \mathrm{v}^{\mathrm{k}} - \mathrm{w}^{\mathrm{k}}\right], \\
\mathrm{v}^{\mathrm{k}+1} &= \operatorname{argmin}_{\mathrm{v}} \frac{\alpha}{2\beta}\|\mathrm{u}^{\mathrm{k}+1} - \mathrm{v} + \frac{1}{\alpha}\mathrm{w}^{\mathrm{k}}\|_2^2 + \mathrm{TV}(\mathrm{v}), \\
\mathrm{w}^{\mathrm{k}+1} &= \mathrm{w}^{\mathrm{k}} + \alpha\left(\mathrm{u}^{\mathrm{k}+1} - \mathrm{v}^{\mathrm{k}+1}\right),
\end{aligned}
\tag{20}
$$

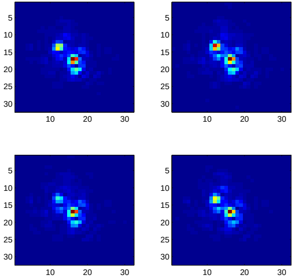
where the second problem is solved with the algorithm by Chambolle (2004).

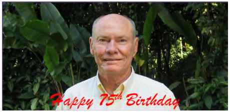


A Binary-Star Example

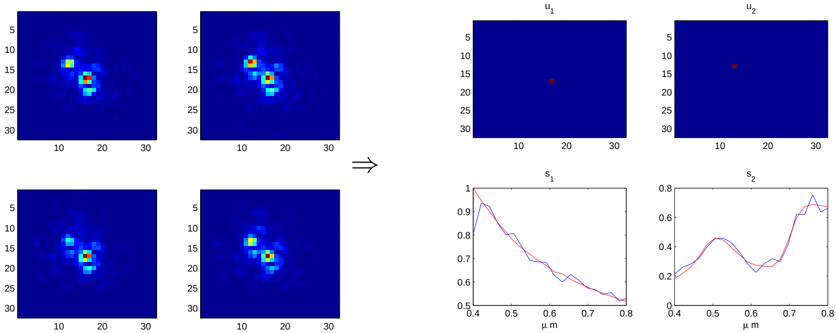


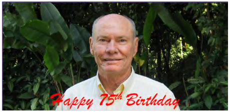
$\Rightarrow$



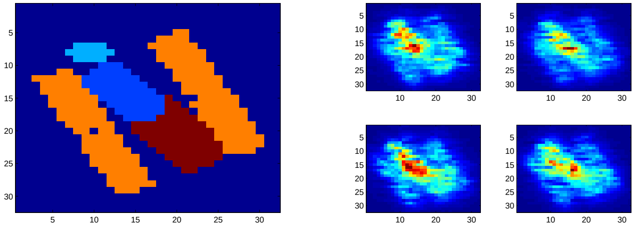


The Binary-Star Example: Reconstructed u and s



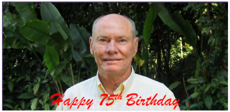


The HST Example: Dense Scene

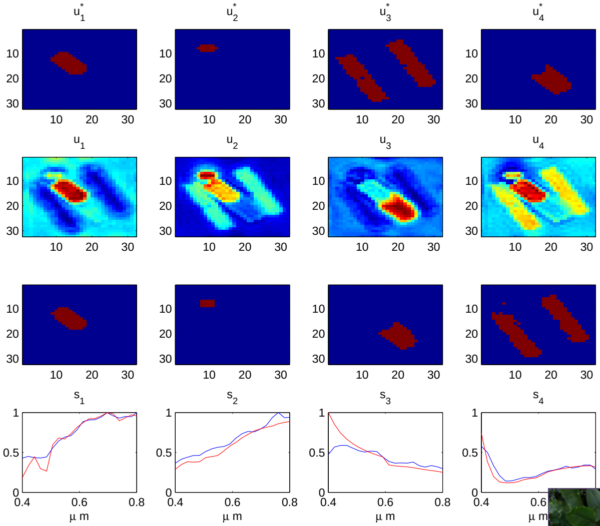


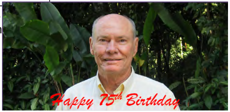
(a) (b)

Abbildung : (a) A spectral band of simulated HST image. (b) The simulated DD-CASSI image of HST through atmospheric turbulence.



The HST Example: Reconstructed u and s





Conclusions

We have proposed a two-step semi-blind deconvolution and spectral unmixing model for reconstructing hyperspectral objects from compressed measurements of blurred hyperspectral images. Even though the posed inverse problem is highly challenging, the model's success relies on two key factors:

1. The snapshot spectral imager effectively reduced the more severe blurring effects at lower-wavelength channels through coding and multiplexing with longer-wavelength channels where blurring is less severe;
2. The decomposed solution model significantly reduces the number of unknowns, and improved the system matrix property to guarantee a better reconstruction success probability.